ON ACHROMATIC IP DESIGN FOR MC

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OUTLINE

- IR as a microscope
- Star point chromaticity issue
- Compensation conception in general
- Compensation equations
- Compensating block structure
- Symmetry of magnetic structure and particle trajectory: chicane, symmetric dispersion, symmetric vertical trajectories, anti-symmetric free horizontal trajectories, symmetric sextupole set
- Two impositions on sextupole field to compensate the star point chromaticity
- Compensation for octupole fringe fields effect
- Tolerances
- Minimal achievable beta-star

Chromatticity impact on star focus

F –***focal parameter of the final focusing block (FFB)***

$$\beta^* = \frac{F^2}{\beta_f} \qquad (\frac{\sigma_f}{\sigma_0})^2 = \frac{\beta^f}{\beta^0} \qquad \beta^0 \approx F$$

Spread of the focus point: $\Delta F = \frac{\partial F}{\partial p}\Delta p \approx F\frac{\Delta p}{p}$

Requirement: $\Delta F << \beta^*$

- MC case at E=2.5 TeV:

$$F \approx 20M \qquad \beta^* = 5mm \qquad \frac{\Delta p}{p} = .3\%$$

Problem: $\Delta F = 6cm$ $\rightarrow$ **Needs compensation!**
(for 750 GeV, as well…)

***Reference star point (FFB optics control) requirement:**

$$\frac{\Delta F}{F} \approx (\frac{\Delta n}{n})^f << \frac{\beta^*}{F} = \frac{F}{\beta^f} \approx (\frac{\sigma^0}{\sigma^f})^2 \approx 2.5\cdot 10^{-4} \text{ at 2.5 TeV}$$

$$\approx 5\cdot 10^{-4} \text{ at 750 GeV}$$

Is this a problem? I don't think so…

Compensation for chromaticity concepts

- Use long section of beam extension for compensations in a preventive way (i.e. insert a compensating block (CB) before the FFB). Leave FFB alone to provide the strongest star focus

- Design chicane or zigzag CB (even design or use a chicane/zigzag channel if necessary)

- Design a symmetrical quadrupole/sextupole lattice of the CB self-compensated for emittance and squared momentum spread parasitic effects of dispersion and sextupoles

- Check a need in compensation for higher order effects (spherical aberrations, the third order forces…)

Expansion of the s-Hamiltonian

$$H_s = -(1+Kx)[\sqrt{(H_t - A_t)^2 - m^2 - (P_x - A_x)^2 - (P_y - A_y)^2} + A_s]$$

Second order (basic) Hamiltonian

$$H_2 = -Kx\Delta p + \frac{P_x^2 + P_y^2}{2} + \frac{1}{2}[(K^2 - n)x^2 + ny^2]$$

The third order Hamiltonian (*bend contributions neglected …****)***

$$H_3 \approx -\frac{1}{2}(p_x^2 + p_y^2)\Delta p - \frac{1}{3}n_s(x^3 - 3xy^2)$$

The fourth order Hamiltonian (*bend contributions neglected …****)***

$$H_4 \approx \frac{1}{8}(p_x^2 + p_y^2)^2 + \frac{1}{24}n''(x^4 - y^4) - \frac{1}{4}n_{oct}(x^4 + y^4 - 6x^2y^2)$$

$$H = H_2 + h; \qquad h \Rightarrow H_3 + H_4$$

Hamilton's equations:

$$p_x' = -\frac{\partial H}{\partial x} = Kq - (K^2 - n)x - \frac{\partial h}{\partial x}; x' = p_x + \frac{\partial h}{\partial p_x}$$

$$p_y' = -\frac{\partial H}{\partial y} = -ny - \frac{\partial h}{\partial y}; y' = \frac{\partial H}{\partial p_y} = p_y + \frac{\partial h}{\partial p_y}$$

Linear equations:

$$x' = p_x \qquad\qquad y' = p_y$$

$$p_x' = -(K^2 - n)\Delta p + K\Delta p \qquad\qquad p_y' = -ny$$

Trajectory equations:

$$x'' + (K^2 - n)x = K\Delta p; \qquad y'' + ny = 0$$

$$x = D\Delta p + \tilde{x}; \qquad \tilde{x}'' + (K^2 - n)\tilde{x} = 0$$

$$p_x' = D'\Delta p + \tilde{p}_x' \qquad D'' + (K^2 - n)D = K(s)$$

General solution for y: $\begin{matrix} y = au + b\upsilon \\ p_y = au' + b\upsilon' \end{matrix}$; $u'\upsilon - u\upsilon' = const \Rightarrow 1/\beta^0$

Similar for $\tilde{x}$ …

Linear IR design

Let $\upsilon(s)$ taken at starting point (arc end) and star point be particle positions, while $u'(s)$ at these points represents the angle:

$$u^0 = 0, \quad u^* = 0; \quad \upsilon^0 = 1; \; \upsilon'^0 = 0; \quad \upsilon^f = 0$$

The characteristic relationships in the IR:

$$u'^0 = \upsilon^0 / \beta^0 = -1/\beta^0; \qquad u^f = k\upsilon^0; \quad \upsilon^* = \upsilon^0 / k$$

$$\upsilon'^f u^f = -\upsilon^0 u'^0 = \upsilon^* u'^*; \quad \upsilon^* = F\upsilon'^f$$

$$u'^* = -(u^f / F) = -ku'^0$$

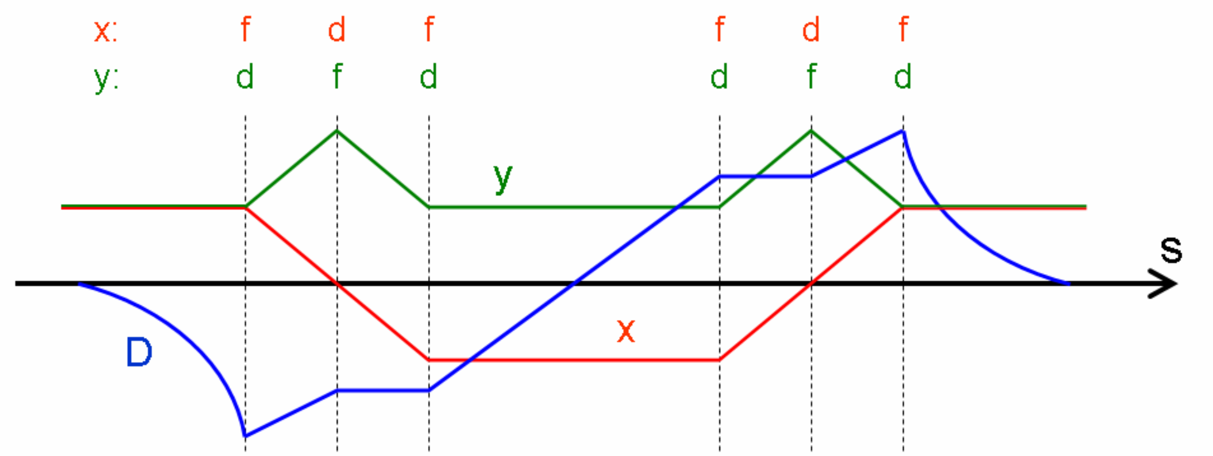
$$\upsilon^* = \sqrt{\frac{\beta^*}{\beta_0}}\upsilon_0; \quad u'^* = \sqrt{\frac{\beta_0}{\beta^*}}u'_0;$$

Note:

$\upsilon(s) << u(s)$ **along the extended beam section, as well as in the FFB**

Compensating block design

- **Insert the CB before the final focus block (FB)!**
- **Design the CB symmetric!**



Symmetry: $n(s)-symmetric$; $K(s), D(s)-antisymmetric$

$u_x(s), u_y(s)-$ **symmetric solutions for** $\tilde{x}, y$ **("parallel" beam gymnastics in CB);**

$\upsilon_x(s), \upsilon_y(s)-$ anti-symmetric solutions (almost zero in CB, as well as along the "parallel" beam sections)

Compensation for aberrations

Non-linear theory

Canonical transformation to amplitudes a and b:

$$\begin{aligned} a &= -\upsilon' y + \upsilon p_y \\ b &= u' y - u p_y \end{aligned}; \quad \{a,b\} = u'\upsilon - u\upsilon' = 1$$

$$a' = -\frac{\partial h}{\partial b} = -\upsilon \frac{\partial h}{\partial y} - \upsilon' \frac{\partial h}{\partial p_y}; \qquad b' = \frac{\partial h}{\partial a} = u \frac{\partial h}{\partial y} + u' \frac{\partial h}{\partial p_y}$$

$$a_y = a_y^0 - \int_0 (\upsilon \frac{\partial h}{\partial y} + \upsilon' \frac{\partial h}{\partial p_y}) ds \equiv a_y^0 + \Delta a_y$$

$$b_y = b_y^0 + \int_0 (u_y \frac{\partial h}{\partial y} + u_y' \frac{\partial h}{d p_y}) ds \equiv b_y^0 + \Delta b_y$$

$$b_y^* = b_y^0 + \Delta^* b_y; \qquad \Delta^* b_y = \int_0^* (u \frac{\partial h}{\partial y} + u' \frac{\partial h}{\partial p_y}) ds$$

Reduction of tuning equations

General tuning requirements:

$\overline{(\Delta^* b)^2} << \overline{b_0^2}$ or $\Delta^* b \Rightarrow 0$

Reduction of the integrands (since υ is very small):

$$\tilde{x} \Rightarrow a_x u_x(s); \qquad y \Rightarrow a_y u_y(s);$$
$$p_x \Rightarrow a_x u_x'(s); \qquad p_y \Rightarrow a_y u_y'(s)$$

<u>Variations of a in integrals can be neglected</u>

Compensation for quadratic aberrations

$$\Delta b_{y2} = \int_0^* (u_y \frac{\partial H_3}{\partial y} + u'_y \frac{\partial H_3}{\partial p_y}) ds \Rightarrow \int_0^* [2n_s(xyu_y - y'u'_y \Delta p] ds = 0$$

$$\Delta b_{x2} = \int_0^* (u_x \frac{\partial H_3}{\partial x} + u'_x \frac{\partial H_3}{\partial p_x}) ds \Rightarrow -\int_0^* [n_s u_x (x^2 - y^2) + u'_x x' \Delta p] ds = 0$$

at: $y \Rightarrow a_y u_y,\ \ y' \Rightarrow a_y u'_y;\ \ \ x \Rightarrow D\Delta p + a_x u_x,\ \ x' \Rightarrow D'\Delta p + a_x u'_x$

Compensation task is then expressed, generally, in 5 conditions:

1) $(2\int Dn_s u_y^2 ds - \int u_y'^2 ds)\Delta p = 0$

2) $(2\int Dn_s u_x^2 ds + \int u_x'^2 ds)\Delta p = 0$

3) $(a_y^2; a_x a_y) \int n_s u_x u_y^2 ds = 0$

4) $a_x^2 \int n_s u_x^3 ds = 0$

5) $(\Delta p)^2 \int (n_s D^2 - nD) u_x ds = 0$

One has to design an anti-symmetric sextupole field, then **conditions 3), 4) and 5) are satisfied automatically!**
Still two "original" conditions to satisfy:

$$2\int Dn_s u_y^2 ds = \int u_y'^2 ds; \qquad 2\int Dn_s u_x^2 ds = -\int u_x'^2 ds$$

Precision required forchromatic compensation control: $\approx 1\%$

Compensation for the cubic aberrations

$$\Delta b_{y3} = \int_0^* \left(y\frac{\partial H_4}{\partial y} + y'\frac{\partial H_4}{\partial p_y}\right)ds$$

$$\Delta b_{x3} = \int_0^* \left(\tilde{x}\frac{\partial H_4}{\partial x} + \tilde{x}'\frac{\partial H_4}{\partial p_x}\right)ds$$

- My examination of all the 4th order effects (including higher order effects of the cubical terms in the Hamiltonian) showed that they all are negligible for high energy colliders (MC, EIC, LHC, ILC…)

Conclusion and outlook

- So, I presume with expression of a confidence that we can design a non-aberrative IP with 5 mm beta-star for MC – at least when developing a green field design…

- Main issue (as to me…): how large the preventive compensator is going to be? The design will show… but I am sure that it will be much shorter than the space usually needed for beam extension before the detector area

- Finally, can an even shorter beta-star be realized? (higher fields, shorter bunches, better the orbit and IP control). These possibilities will pay back to ease the efforts on transverse emittance reduction by REMEX. It seems to deserve some investigation efforts…

Thank you!